Discounts for Illiquid Shares and Warrants: The *LiquiStat*™ Database of Transactions on the *Restricted Securities Trading Network*

Espen Robak, CFA[*]

Pluris Valuation Advisors White Paper

Original Draft: September 19, 2006

This Draft: January 22, 2007[†]

Abstract

Studies of restricted securities private placements have explained the difference between the price of stock sold to private investors and the issuer's contemporaneous stock price in the market with factors such as the information asymmetry between issuer management and buyers, the possible impending financial distress of the issuer, or control and monitoring services provided by the buyers. In this paper, the implications from the LiquiStat™ database of investor-to-investor trades in restricted securities are explored. The pricing discounts observed in these trades are proposed to be entirely due to the illiquidity of the shares sold. The LiquiStat data also provides evidence of investor preferences with regard to non-traded, illiquid warrants and options. Factors considered possibly related to the magnitude of the discounts are analyzed in the cross-sectional regression model.

---

[*] President, Pluris Valuation Advisors LLC. Pluris Valuation Advisors provides valuations of restricted stock, warrants, stock options, and other restricted or illiquid securities, as well as minority, non-controlling interests in businesses, real estate holding companies, securities holding companies, and similar interests for portfolio valuation, financial reporting, estate, gift, or income tax, and litigation purposes. Questions or comments are welcome at erobak@plurisvaluation.com.

[†] An earlier draft previously published. See Robak, E. (2007) Lemons or Lemonade? A fresh look at restricted stock discounts, Valuation Strategies, January/February 2007.

Contents

Appendices

1. Introduction

In the financial literature, liquidity studies typically investigate subtle differences in liquidity between asset classes or trading markets. Appraisers, on the other hand, often refer to a fixed standard of marketability, against which other securities should be measured. For common stock, the standard is "cash in three days." Private equity or restricted shares of public companies, then, are valued at a "discount for lack of marketability."[1] No one doubts that liquidity affects security prices, but the extent of the liquidity-effect is hotly debated. This white paper provides a brief overview of the literature on restricted stock private placement discounts, discusses the shortcomings of traditional private placement studies, proposes an alternative data-set, and suggests a possible way forward.

There is no commonly-accepted theoretical model of liquidity, although a few have been proposed. Conceptually, investors value liquidity and would rather hold liquid assets than illiquid ones. And investors are more concerned about being "locked in" with an investment the more likely it is to lose value during the period of illiquidity. Two main kinds of data have been suggested for determining liquidity discounts for equity: restricted stock private placement data and pre-initial public offering data. The latter is beyond the scope of this white paper, which will focus on restricted stock data.[2]

The lack of generally accepted methods for measuring the value of liquidity, or the value loss from illiquidity, is severe enough with respect to common equity. But it is worse still with respect to stock options or warrants. While several studies of illiquid share private placements have been available for appraisers, academics, and analysts – whatever their shortcomings – no such studies of transactions in illiquid options or

[1] Throughout this paper, the terms "liquidity discounts," "marketability discounts," and their equivalents are used synonymously.

[2] While beyond the scope of this white paper, the highly-persuasive arguments against the pre-IPO approach in the Tax Court's en banc decision in *McCord [McCord v. Comm'r, 120 T.C. No. 13 (May 13, 2003).]* should be noted by appraisers that use this method. The *McCord* decision sets a very high hurdle for anyone relying on the pre-IPO data as a primary approach to liquidity discounts in a tax case.

warrants have ever been available until the release of data from the LiquiStat™ database. This white paper will begin to explore the factors that drive illiquidity discounts for non-traded illiquid warrants and options.

2. Private Placement Discounts

When companies issue equity in private placements, they typically do so at a discount. In the financial literature there are four common explanations advanced for this discount: illiquidity effects, control and monitoring effects, capital scarcity effects, and information asymmetry effects. The illiquidity effect explanations, while understandably popular in the tax and accounting literature, have not been the subject of much attention in the (academic) financial literature. Silber studied illiquidity discounts based on restricted stock private placements, but did not discuss other possible explanations for the discount.[3] The study of Bajaj, et. al., is unique in this connection, as they attempted to separate the illiquidity effect from other effects.[4] Since this white paper focuses on liquidity discounts, the other explanations will be referred to as "alternative" explanations.

Control and Monitoring. Based on 73 private placements of both registered and unregistered stock by relatively large, NYSE-traded companies, Wruck found a 12 percent average discount for the unregistered shares.[5] Regressing the change in firm value on indications of ownership concentration, she found some support for her theory that the issuer expected to benefit from the active role that large investors take in management, at both low and high levels of ownership concentration. She concluded that the discount could be partially explained as compensation to the buyers for their monitoring activities.

Capital "Scarcity." Lee & Kocher argue that a major motivation for firms issuing equity privately is that they need an infusion of capital but cannot raise funds through the

[3] Silber, W. (1991) Discounts on restricted stock: The impact of illiquidity on stock prices, Financial Analysts Journal 47, 60-64.
[4] Bajaj, M., and D. Denis, S. Ferris, and A. Sarin (2001) Firm value and marketability discounts, Journal of Corporation Law, 27, 89-115.
[5] Wruck, Karen H. (1989) Equity ownership concentration and firm value: Evidence from private equity financings, Journal of Financial Economics, 23, 3-28.

traditional public markets due to their financial condition.[6] In other words, the public offering window is effectively shut. Chu, et. al., argue that part of the private placement discount may be compensation to investors for their willingness to contribute capital to firms that are showing signs of financial distress (e.g., negative earnings).[7] Thus, to paraphrase the argument, under pressure of both capital scarcity and financial distress, management is willing to issue new equity at discounts that may sometimes be "too deep." The theory of how capital scarcity may or may not affect liquidity discounts is not well developed. However, studies have shown a connection between the discount and both negative earnings and bankruptcy risk.

[6] Lee, H. W., and C. Kocher (2001), Firm characteristics and seasoned equity issuance method: private placement versus public offering, The Journal of Applied Business Research, 17, 23-36.

[7] Chu, S-H., G. Lentz, and E. Robak (2005), Comparing the characteristics and performance of private equity offering firms with seasoned equity offering firms, Journal of Economics and Management, 1, 57-83.

3. Information Asymmetry: the "Lemons" Problem

Akerlof, in a Nobel prize-winning paper, described the Lemons Problem as a combination of information asymmetry and adverse selection.[8] Noticing the wide disparity in price between new cars and cars that have just left the showroom, he suggested two differences between the situations: the amount of information the seller has about the car and the selection of cars being sold. When buying a new car, the buyer has almost exactly the same amount of information about that particular car as the seller (namely, very little). Also, the selection on any given lot can be assumed to be a random draw from the population of that brand. In other words, the seller has no incentive to sell a particular vehicle because he or she has information that makes him question its value, and also, because the selection of vehicles have not been on the road yet, the process of "sorting" good from bad cannot have begun yet.

When buying a used car, on the other hand, the buyer knows less about the car than the seller does (information asymmetry). The seller has owned and driven the car for months or years. The buyer knows this and also knows that the seller is more likely to sell if he knows that it is a bad car – a lemon (adverse selection). Akerlof showed that the buyer would tend to discount his offer price more than he would if information asymmetry was not a problem.[9] Myers & Majluf found that information asymmetry and adverse selection combined may cause firms to forego otherwise-attractive investment opportunities if they need outside capital to pursue them.[10] Investors know, or fear, that managers will want to raise capital if the stock price is "high" and avoid raising capital if the price is "low." Therefore, they bid down the price of companies raising money.[11] This, in turn,

[8] Akerlof, G. (1970), The market for "lemons": quality uncertainty and the market mechanism, Quarterly Journal of Economics, 488-502.

[9] Akerlof further showed that, under certain conditions, a vicious cycle can result, wherein buyers steadily ratchet down their bid prices and sellers steadily reduce the quality of their offerings, to the point where, in the extreme case, a market may cease to exist.

[10] Myers, S., and N. Majluf (1984), Corporate financing and investment decisions when firms have information that investors do not have, Journal of Financial Economics 13, 187-221.

[11] The argument assumes that management will work to favor existing shareholders (who, after all, gave them their jobs in the first place) or that management *is* among the existing shareholders and, thus, will attempt to minimize the dilution from any capital-raise.

exacerbates the adverse selection problem. Empirical studies have shown that companies that issue shares in secondary public offerings tend to see their stock prices drop on announcement, consistent with the Lemons Problem theory.[12]

Hertzel & Smith, in an extension of Myers & Majluf, explained private placement discounts as compensation to investors for costs they incur to reduce asymmetries of information.[13] They analyzed a sample of 106 private placements from the 1980-87 period with an overall average discount of 20 percent and a lower average discount for the registered shares. Testing their theory, they regressed the discount on measures associated with increased uncertainty about firm value, such as evidence of distress or high market-to-book ratios. However, they almost completely ignored the issue of liquidity as a contributor to the discount.[14]

As an extension of the Hertzel & Smith analysis, Bajaj, et. al., studied a sample of private placements that included both registered and unregistered shares, with the explicit goal of providing separate estimates for the contributions of liquidity and information asymmetry, respectively. They analyzed 88 private placements, with an overall average discount of 22 percent. Because the registered shares sold at significant discounts, albeit lower than the unregistered shares, the authors concluded that the private placement discount had to be caused by factors other than illiquidity. They assumed, in other words, that these "shares can be transacted freely, and the fact that the firm was publicly traded meant there was a ready market for these shares."[15] They regressed the discount on a number of variables, all associated with information asymmetry or control issues, plus only one variable associated with liquidity (the dummy variable indicating that registered

[12] See, for example, Asquith, P, and D. Mullins (1986), Equity issues and offering dilution, Journal of Financial Economics 15, 61-89.
[13] Hertzel, M, and R. Smith (1993), Market discounts and shareholder gains for placing equity privately, Journal of Finance, 48, 459-485.
[14] *Ibid*, p. 480. The impact of liquidity, or lack thereof, was discussed in a footnote to the paper. Noting that there are institutional investors that "do not value liquidity highly," the authors dismissed the idea that the incremental discount for the unregistered shares was due entirely to the illiquidity of the shares, but did not investigate further.
[15] Bajaj, et. al., *supra*, p. 107.

stock was sold).[16] The authors concluded, on the basis of this analysis, that the only portion of the private placement discount caused by lack of liquidity was 7.2 percent, on average. However, they also opined that the appropriate discount to apply for lack of marketability when determining the fair market value of minority interests in operating companies is the full private placement discount, or 22 percent on average.[17]

[16] It is interesting to note that this dummy-variable was used by Hertzel & Smith as a variable related to information asymmetry (because the "quality-signaling" effect to the market would be greater if the private investors are locked in through owning restricted stock). There is certainly nothing *per se* wrong with this. As Myers & Majluf put it, "a full description of corporate financing and investment behavior will no doubt require telling several stories at once." The private placement data is clearly quite "malleable" in the sense that the same sample of transactions can tell us many different stories.

[17] Bajaj, et. al., *supra*, p. 114. "In our opinion, when valuing an operating company that is privately held (or its securities), the appropriate benchmark for discounts is provided by the total private placement discount or the discount observed in the acquisition approach." Note that the authors claim the "total" private placement discount, which might include both the "registered" and unregistered shares (average: 22%) should apply. However, if the goal is to find the right discount for fully-illiquid securities, it clearly makes the most sense to exclude any shares that were sold with registration pending or with registration rights from the analysis. This portion of their sample had an average discount of 28%.

4. Problems with the Private Placement Studies

The final conclusion of Bajaj, et. al. – that the full private placement discount should apply to the value of private equity – is highly debatable. The standard of value relevant to readers of their paper is "fair market value" in the way this term is generally interpreted in a tax context. In the fair market value context, the identities of buyer and seller are supposed to be unknown, similar to the way stocks trade in the public markets.[18] In other words, they are supposed to be unknown to the valuation analyst. Thus, any analysis of value that determines a price appropriate for the securities only in certain types of transactions or only between certain kinds of buyers or sellers is automatically suspect. The goal, at any rate, is to find the "consensus" price or the price which would be acceptable to the greatest diversity of buyers or sellers and in a variety of different transaction types. In particular, if we assume that the Lemons Problem is a serious consideration which weighs heavily on the transaction decisions of buyers in the PIPEs (Private Investments in Public Equity) market, the prices paid in such transactions would likely not apply in different circumstances.

The best reflection of the fair market value of a security would be an arm's-length trade between two investors that are anonymous and unaffiliated with the issuer. And in such a trade there would be no systematic tendency for information asymmetry to bias prices in one particular direction because there would be no Lemons Problem.[19] In fact, all of the alternative discount explanations are based on the assumption that private placements are sold at prices that are different from those at which an investor would buy the same shares with the same restrictions in ordinary market transactions. In other words, the

[18] Courts have consistently held that the buyers and sellers assumed by the definition are "hypothetical" and have rejected analyses where the identities of buyer and seller are important to the outcome.

[19] Note that it is an oversimplification to state that information asymmetry holds between buyers and sellers in normal market transactions: it almost never actually does. However, in normal market transactions, neither buyers nor sellers have any particular reason to suspect that the *other side of the deal* has more information than they do. (Sometimes it might be the buyer who has the most information; sometimes it might be the seller who has more information, with no particular tendency for it to go one way or the other). Also, as buyers and sellers have the same information, on average, there is no adverse selection problem. In other words, the seller is not selling because he is an insider and has access to material nonpublic information about this particular stock.

peculiarities of the private placement process produce pricing-effects that are unrelated to the market value of the stock.[20] If we were to accept the Bajaj, et. al., analysis in theory, the logical conclusion would be to eliminate any portion of the private placement discount truly caused by factors other than illiquidity from the total marketability discount.[21]

However, we should not accept any of these explanations without further scrutiny, especially since the studies tell very different stories with the same data and almost the same analysis. There are two inherent weaknesses of the private placement studies: (1) the lack of measurable parameters that are exclusively associated with either of the phenomena the researcher wishes to analyze: control and monitoring, information asymmetry, or the alleviation of capital scarcity; and (2) the impossibility of establishing two distinct data-sets, one completely liquid and one completely illiquid.

"Alternative Explanation" Parameters. It would be easy to tell stories using almost any of the variables analyzed in the private placement studies as an indicator for almost any one of the four discount explanations. As an example, Bajaj, et. al., propose that all of the following four variables are associated with information asymmetry and not with illiquidity: fraction of total shares placed, stock price volatility, the z-score, and total placement proceeds. However, the greater the fraction of total shares placed the more difficult it is to re-sell the shares after the placement. Thus, fraction of total shares placed is also associated with illiquidity. Also, in Wruck's telling, the fraction of shares placed is associated with increased ownership concentration and part of the control and monitoring story. Likewise, stock price volatility is associated with the effect of illiquidity: the greater the volatility, the greater the chance that a security will lose value.

[20] And, to be sure, the typical arm's-length transaction analysts envision when attempting to determine "fair market value" of a fractional interest in a privately held operating company does not involve providing a huge cash infusion to the company, often to the tune of 10-20 percent of its market value, radically altering its balance sheet, growth outlook, and other financial and operating characteristics.

[21] This analysis should apply in almost any appraisal context, except a valuation involving the issuance of new shares for new capital to the issuing firm or a trade between an inside seller and an outside buyer. This level of knowledge of the identities of buyer and seller is incompatible with both the "fair market value" and the "fair value" standards.

The exact same goes for the z-score: a lower z-score and higher bankruptcy risk makes investors crave liquidity more. Finally, total placement proceeds are greater the greater the market value of the firm. Larger firms are less risky, which reduces the discount for illiquidity. Reasonable arguments can support several explanations for the discount. They are all plausible stories; none of them dominates.

Registration Variables. A more significant problem is the inability of the registration dummy variable to capture the entire difference between liquid and illiquid securities. Consider, first, the fact that in Hertzel & Smith's and Wruck's telling the registration variable is part of the information asymmetry and control and monitoring stories, respectively. Secondly, consider the facts of life for investors in private equity: there is no such thing as privately placed stock that is completely liquid. The SEC discourages private sales of already-registered shares. Thus, the stock flagged as "registered" in the private placement studies was probably not registered *prior to* the placement (Hertzel & Smith states the shares were either registered or sold with registration "pending"). It is also conceivable that some of the shares placed merely had registration *rights*.[22] Most importantly, the average fraction of total shares placed was more than 15 percent on average for the studies reviewed here. This large block size is enough to induce liquidity constraints on resale after the private placement, regardless of registration status.

The alternative explanations for the private placement discount are well-supported by both logic and empirical data, and they likely explain at least part of the discount. However, the studies cannot accurately *measure* the discount portion caused by factors other than illiquidity. There may also be facets to the pricing of private placements that are still unexplored. For example, private investors sometimes get board seats or other valuable elements of control over the issuer. They might pay more because of these additional features. This, then, would tend to reduce the observed private placement

[22] Shares sold with registration "pending" are still subject to SEC review of the registration statement, which can be lengthy and might never result in an effective registration statement. Shares sold with either demand or "piggyback" registration rights depend on the issuer's willingness and ability to live up to its promise to register the shares. In either case, the period of illiquidity can be lengthy and uncertain.

discount. In other words, the average illiquidity "portion" of the discount might be higher than the average total discount observed in the studies.[23]

In summary, when comparing the "pure" illiquidity discount with the private placement discount, it seems that the former may be higher, lower, or the same as the latter, depending on facts and circumstances. And it is very hard to tell which way it goes for the average issuer or the average transaction, by reviewing the private placement data alone.

Post-Deal Performance

If investors are adequately compensated for their private placements investments, they should earn above-market (or at least market) returns on their investments during the period of illiquidity. However, this does not appear to be the case. Hertzel, et. al., find that companies that issue equity privately tend to under-perform the market indexes, on a risk-adjusted basis. [24] This is unexpected, since the announcement effect is positive.[25] Chu, et. al., (in a paper coauthored by this author) also found that investors in private placements did poorly.[26] In this sample, the private investors underperformed the risk-adjusted index, even when taking the discount into account. Due to the higher average betas of private placement firms, the significant discounts taken "may nonetheless be too small to compensate investors on a risk-adjusted basis."[27]

Clearly, the private placement process has facets, beyond just illiquidity, that affect discounts. The solution, or at least part of the solution, might be to take a look at the

[23] So, in a simplified example, if a private placement was done at a 30% discount from market price, while the purchaser got control features worth 15% (of the same base), then, ignoring all of the other discount explanations, the "true" illiquidity discount was 45%.
[24] Hertzel, M., M. Lemmon, J. Linck, and L. Rees (2002) Long-run performance following private placements of equity, Journal of Finance, 57, 2595-2617.
[25] Both Wruck and Hertzel & Smith found that the announcement effect was positive, i.e., when companies announce a private placement, the stock price showed significant abnormal returns across the announcement window.
[26] Ibid, at p. 74.
[27] Ibid, at p. 81.

pricing of restricted stock in investor-to-investor trades, not involving the issuer or an affiliate of the issuer and not raising capital for the issuer. This we will do below.

5. Theoretical Models

The illiquidity discount data can also be used to test the available theoretical models for illiquidity discounts. The three models we test are developed by Longstaff, Finnerty, and Tabak.

First, Longstaff proposes a model of the "upper bound" on the discount, based on the well-known pricing formula for a lookback option.[28] The discount, in this model, is purely a function of the time remaining until the end of the illiquidity period and the volatility of the stock. The model assumes that investors have perfect market timing ability. The formula for the percent discount (d) is given below:

$$d = \left(2 + \frac{\sigma^2 T}{2}\right) N\left(\frac{\sqrt{\sigma^2 T}}{2}\right) + \sqrt{\frac{\sigma^2 T}{2\pi}} e^{\left(-\frac{\sigma^2 T}{8}\right)} - 1, \text{ where}$$

σ = the volatility of the stock; and T = period of illiquidity remaining. The volatility used in the formula, as in standard option pricing formulas, is the annualized standard deviation of return on the stock.

The Longstaff formula calculates the largest discount possible for a given security in a market of rational investors, assuming that illiquidity was the only thing causing the discount. Longstaff attempted to test the model against average discounts from restricted stock private placement studies. He found that the observed discounts were close to the upper bound, assuming that the average volatility for private placement companies are in the 25-35 percent range and that the average holding period for restricted stock in the studies was 2 years. However, for the older restricted stock studies (mostly performed during the 1960s – 1980s), the average effective holding periods may have been higher than 2 years. More importantly, the average stock price volatility for companies placing

[28] Longstaff, F. (1995) How much can marketability affect security values?, Journal of Finance, 50, 1767-1774.

equity privately was almost certainly higher than the 25-35 percent range during this period. More recent studies show that companies placing stock privately are more volatile than the average public company.

Finnerty proposes a model for the discount based on the pricing-formula for average-price put options, also known as "Asian" options.[29] This model does not assume any special market timing abilities on the part of the investor. The model derives an upper bound for the discount, as follows:

$$d = e^{(r-q)T} N\left(\frac{r-q}{v}\sqrt{T} + 1/2 v\sqrt{T}\right) - N\left(\frac{r-q}{v}\sqrt{T} - 1/2\, v\sqrt{T}\right), \text{ where}$$

$$v^2 = \sigma^2 T + \ln\left(2e^{\sigma^2 T} - 2\sigma^2 T - 2\right) - 2\ln\left(e^{\sigma^2 T} - 1\right), \text{ and where}$$

r = the risk-free rate of return and q = the dividend yield.

This model, at any given level of stock price volatility, results in a straight-line relationship between the period of illiquidity and the discount. Finnerty tests the model on "implied" illiquidity discount data based on a sample of 101 private placements and finds that it fits the data rather well for stocks of medium volatility (σ between 30 and 120 percent), but not for high or low volatilities.[30] The model derives significantly lower "upper bound" discounts than the Longstaff model.

Tabak's model assumes that the stock cannot be hedged.[31] The model estimates the value loss (i.e., discount) resulting from the illiquidity of the stock, combined with this "no hedging" constraint. The model is based on the time of illiquidity, the volatility of each

[29] Finnerty, J. (2003). The Impact of Transfer Restrictions on Stock Prices, Fordham University working paper, draft dated June 2003, available at www.fordham.edu.
[30] *Ibid*., at p. 29.
[31] Tabak, D. (Undated), A CAPM-based approach to calculating illiquidity discounts, Nera Economic Consulting; unpublished white paper, available at www.nera.com.

security, and broad measures of market risk premiums and volatility. The model is shown below:

$d = 1 - e^{-\varphi\theta T}$, where

$\varphi = \sigma^2/\sigma_M{}^2 - \beta$, and where

σ_M = the volatility of the market and θ = the market risk premium.

The Tabak model derives discount indications that are greater than the Longstaff "upper bound" for moderate-to-high volatility stocks. "This would indicate the need for caution in applying this analysis to highly volatile stocks, because the results could exceed the theoretical maximums suggested by other papers."[32]

[32] Tabak, *supra*, p. 16.

6. Restricted Stock and Private Placements

There are three kinds of data reviewed in this white paper: the aforementioned private placement studies, which analyze private placements of both restricted and registered shares; restricted stock private placement studies, which exclude registered or soon-to-be registered shares; and pure investor-to-investor restricted stock transaction data (not involving private placements). The restricted stock private placement studies are well-known from the tax and accounting literature. Over more than a thirty-year span of time, the various articles on such transactions (i.e., private placement samples that exclude, or attempt to exclude, registered shares) have indicated average discounts approximately in the range from 20 percent to 35 percent, with somewhat larger discounts more common in private placements in the 1960s through the 1980s and average discounts in the 20-25 percent range more typical in later years, especially after the SEC reduced the holding period necessary to claim the "4 (1)" exemption in 1997 (see sidebar below).

SIDEBAR

The Evolution of Rule 144 - Persons Deemed Not to Be Engaged in a Distribution and Therefore Not Underwriters

The key provision of Rule 144 is to require a holding period before a purchaser of restricted stock can sell it in the public markets, originally two years. Furthermore, after the holding period, restricted stock sales to the public are subject to the volume limit provisions, and to certain manner of sale, reporting, and notice requirements of the Rule. The Rule has been amended several times since its promulgation in 1974. The most important amendments are noted below:

1983: Addition of Rule 144 Paragraph (k)

Rule 144 (k) allows anyone not affiliated with the issuer to sell the entire block held after a period of three years. The question of whether or not a shareholder is an affiliate is a "facts and circumstances" test. Generally, anyone who owns less than 10 percent of the issuer's stock, does not have a board seat or any management responsibilities, and does not have any control in-fact would not be considered an affiliate of the issuer.

1990: Allowing Holding Period "Tacking"

This amendment allowed anyone who purchases stock from a non-affiliate of the issuer to "tack" the holding period of the previous owner to his own. Prior to 1990, the holding period would "restart" whenever restricted stock was transferred in a private sale, such as the "4 (1-½)" exemption sales noted above. The 1990 amendment greatly improved the marketability of restricted stock.

1997: Shortening the Holding Period

In 1997, the initial holding period was reduced from two to one year and the holding period for non-affiliates to take advantage of Rule 144 (k) was reduced from three to two years.

Most of the studies, in other words, pertain to restricted shares subject to an almost completely different regulatory framework than the one that exists today (the most comprehensive of the earlier studies, the SEC's Institutional Investor study from 1967, was performed before Rule 144 became effective). As a result, their conclusions should be applied to restricted shares today only with extreme care and adjusting for the differences in liquidity for restricted stock today and in the past. The studies also generally show higher discounts for riskier issuers and for larger fractions of total shares placed.[33]

The key provision of Rule 144 is to require a holding period before a purchaser of restricted stock can sell it in the public markets. The holding period is currently one year (was two years prior to April 1997). This provision is necessary for the seller to show that he is not acting as an "underwriter" and, thus, is eligible for the 4(1) exemption, which is meant to apply only to persons who are not participating in a distribution of shares to the public. Furthermore, after the holding period, restricted stock sales to the public are subject to the volume limit provisions of the Rule. During any three month period a restricted stock holder cannot sell more than the greater of one percent of total shares outstanding or the weekly average trading volume during the four weeks prior to the sale. Also, if the issuer at any time fails to file all of the financial statements required for public companies, the 4(1) exemption is not available to its shareholders until it is again-current on its filings. In other words, restricted stock holders will "stuck" – even if they have held their shares for more than a year – if the issuer fails to file timely its 10Ks, 10Qs, or other required filings. Furthermore, the seller has to comply with certain manner of sale, reporting, and notice requirements of the Rule. The Rule has been amended several times since its promulgation in 1974.

[33] See, for example, Robak, E., and L. Hall (2001) Bringing Sanity to Marketability Discounts: A New Data Source, Valuation Strategies, July/August 2001.

SIDEBAR

Privately-Negotiated Resale Transactions – The Section 4(1-½) Exemption

Restricted securities may be resold in privately-negotiated transactions if such transactions comply with an exemption under the Securities Act. Sellers and buyers in privately-negotiated transactions (other than Rule 144A transactions) generally utilize the so-called "Section 4(1-½) Exemption". The Section 4(1-½) Exemption is a hybrid exemption that was derived from Section 4(1) of the Securities Act, which provides an exemption for resale transactions "by any person other than an issuer, underwriter or dealer," and Section 4(2) of the Securities Act, which provides a private placement exemption for issuers. The Section 4(1-½) Exemption contemplates a private resale that is similar to an issuer's Section 4(2) sale. As the SEC has noted, "This is a hybrid exemption not specifically provided for in the 1933 Act but clearly within its intended purpose" (SEC Act Release No. 6188, 1980 WL 29482).

There are certain "best practices" associated with the Section 4(1-½) transfers such as (i) arranging to have a legend placed on the securities alerting the buyer as to the restricted character of the securities, (ii) arranging to have the issuer issue a "stop transfer order" to the transfer agent for the securities to prevent a subsequent resale without an opinion by counsel as to the legality of such resale, (iii) making basic inquiries into the identity of the buyer, and (iv) securing a representation from the buyer that states the buyer is aware of the restrictive character of the securities and such buyer intends to hold the securities for investment. A typical resale transaction for restricted securities of a public company involves the seller and buyer agreeing to economic terms and then entering into definitive documentation. The definitive documentation will usually consist of the following:

- purchase agreement between the parties;
- buyer's representation letter;
- seller's representation letter;
- opinion letter from company counsel; and
- escrow agreement (if the parties choose to place the restricted security and the purchase funds

Once definitive documentation has been executed by the parties, the executed opinion letter and the accompanying securities instrument and transfer deed (e.g., stock certificate and stock power), are sent to the company's transfer agent, along with a written instruction letter to have the stock certificate reissued in the name of the buyer. Assuming that the documentation is in order, and the issuer has not objected to the transfer, the transfer agent will cancel the stock certificate and reissue a new certificate. During this process, the buyer's cash payment is typically held in escrow pending completion of the transfer.

Note: Excerpted, with permission, from http://www.restrictedstockpartners.com/3_b.asp

There are, generally, two ways to sell securities legally in the United States: file an effective "1933 Act" registration statement and sell the stock to the public or sell the stock in reliance on an exemption from the registration requirement. The "4 (1)" exemption allows for resale to the public of privately issued restricted stock and is available to investors who comply with Rule 144. Such sales effectively end the period

of illiquidity. The other major exemption of interest to holders of restricted stock is the so-called "4 (1-½)" exemption. This allows the holder to sell his restricted stock privately, under certain tightly controlled circumstances, to an accredited investor (see sidebar). Sales under this exemption are the subject of the next section.

7. Empirical Analysis – Restricted Stock

Transaction Sample

The data analyzed herein is from the LiquiStat™ database of private sales transactions. The database is created by Pluris Valuation Advisors.[34] LiquiStat contains transactions facilitated by Restricted Stock Partners from April 2005 to December 2006.[35] The buyers and sellers tend to be hedge funds, institutions, or other accredited investors. The data-set for this analysis was 61 trades in restricted common equity, with no warrants attached and nothing changing hands except cash for stock. Both buyers and sellers were the beneficiary of due diligence performed by the firm facilitating the sale, and by legal counsel. In particular, the ownership history of the stock was known, which would allow both buyer and seller, in each case, to estimate with precision the number of days of illiquidity remaining for each block of stock.

The due diligence performed for each trade, however, was limited to information about the rights, preferences, privileges, and restrictions, as well as the ownership history, of each block of restricted shares. In addition, we can assume that both buyer and seller had access to public data on the securities, including trading price and volume histories, stock price volatilities, and similar information typically instantly available to sophisticated investors. However, the investors were not affiliated with the issuer and did not have access to any material non-public information about the issuer.[36] Until each transaction is

[34] The LiquiStat database is a compilation and analysis of restricted securities trading data, licensed to Pluris Valuation Advisors LLC from Green Drake Capital Corp., its affiliate. The database currently spans 140 transactions (mostly warrants) over approximately 18 months. More information is available at www.plurisvaluation.com/liquistat.

[35] Restricted Stock Partners, of New York, New York, is a division of Green Drake Capital Corp., member NASD/SIPC. Restricted Stock Partners has created the Restricted Securities Trading Network (RSTN). The RSTN is believed to be the largest trading network for restricted securities anywhere, with more than 200 institutions and accredited investors as members. More information is available at www.restrictedsecurities.net.

[36] Note that this is contrary to the private placement process, where such non-public information is routinely provided to prospective investors. The fact that management often provides investors with non-public information during the due diligence process before a private placement, however, should not be interpreted as implying that this information exchange eliminates information asymmetry from the

priced, buyers and sellers were unknown to each other: the only thing they knew about each other was whether or not the seller was an affiliate of the issuer. In the sample reviewed here, there were no affiliate sellers. In other words, we can assume information symmetry between buyer and seller.[37] See Appendix 3 for more on how the RSTN process works.

Each RSTN transaction is made through a time- and date-stamped restricted securities order. This allowed Pluris to determine with precision the market reference price at the exact point each restricted stock trade was made. This is in sharp contrast with private placement studies where the time of pricing of each transaction is almost always unknown.[38] At a minimum, the resulting uncertainty regarding the market reference price introduces random "noise" to the private placement discount samples.

Table 1 -- LiquiStat database descriptive statistics

	Shares-to-Volume	*Market Cap ($m)*	*Days Left*	*Market Price*	*Volatility*	*Discount*
Mean	3.1x	325	143.7	$9.74	87.6%	32.8%
Standard Deviation	7.5x	285	106.8	8.81	36.0	14.9
Minimum	0.0x	7	3.0	0.29	35.2	10.1
Maximum	56.5x	1,079	475.0	33.30	211.2	65.7
1st quartile	0.3x	87	55.0	$3.17	69.1%	19.1%
median	1.0x	247	120.0	7.80	77.2%	34.6%
3rd quartile	2.7x	493	211.0	14.45	87.9%	44.0%

process. Insiders will always know more than outside investors. Also, outside investors will always suspect that management's presentation of facts is selective and designed to put a positive "spin" on current operations and future prospects.

[37] Information asymmetry would tend to hold, *on average*.

[38] The private placement and restricted stock studies solve this problem in different ways. The exact date of the private placement is often unknown. And even if it is known it would be difficult to be certain when the placement was actually priced (it may have been priced at any point in time before closing). The best solution, therefore, may be to use some average price for the month or week of the transaction. This is the approach used by the FMV restricted stock study, the most comprehensive recent database available (continuously updated data). Alternatively, many of the private placement studies use a set date after the announcement date (assuming this date can be ascertained with precision), usually T+10, following Hertzel & Smith. This method, it is assumed, will factor in the expected stock price appreciation on announcement (the positive announcement effect). Needless to say, all of these methods are imprecise ways of determining the market reference price for each placement. Some of the errors may cancel themselves out across large samples. However, the added "noise" will tend to reduce the utility of smaller samples.

See Table 1 for a description of the LiquiStat sample. The average issuer market capitalization is $325 million. The average fraction of total shares placed is 0.47 percent. The trading markets for the stocks in the database are not liquid (average daily trading volume around 190,000 shares) compared with those of the typical large-cap public company. However, the shares-to-volume ratio is still only 3.1x on average. The average volatility of the sample is 89 percent. The average number of days left before the shares sold became available to trade in the public markets is 138, while the interquartile range of days left extends from 55 days to 203 days.

The average discount for the LiquiStat database is 32.8 percent, as shown in Table 1. This is greater than the average discounts seen in most private placement and restricted stock studies. The standard deviation of the sample is 14.9 percent. The median discount is 34.6 percent and the interquartile range extends from 19.1 percent to 44.0 percent. A comparison of discount statistics of the LiquiStat sample and comparable data from private placement and restricted stock studies is presented in Table 2. The average discount for the Finnerty sample was 20.1 percent.[39] The average discount for the Bajaj, et. al., sample is 22.2 percent.[40] The average discount for the FMV sample is 22.0 for the 1980-2005 period, 21.6 percent for the 1997-2005 period and 14.6 percent for the 2002-2005 period. Because the distributions are not normal, it makes sense also to compare the medians – which also appear greater for the LiquiStat sample. Finally, based on the Wilcoxon rank sum test, the difference between the LiquiStat and the FMV discount samples is significant at the 1 percent level, for all three time periods analyzed in the FMV sample.[41]

[39] Ibid., p. 14. Finnerty also reports discounts measured relative to the stock price 10 days prior to the announcement. The average discount for the "10 days prior" measure was 18.41 percent, which is not statistically significantly different from the "day prior" measure.

[40] Ibid., p. 107. Bajaj also reports that the average discount for registered issues was 14 percent and the average discount for unregistered issues was 28 percent.

[41] This test was not possible for the Finnerty or Bajaj data, as it was not available. The FMV data is available online.

Average gross proceeds and the average fraction of total shares placed are both smaller, by an order of magnitude, than transaction sizes in all of the previously published studies. This is expected, as the transactions herein are between individual investors. The private placement studies, on the other hand, analyze large corporate transactions raising significant amounts of new capital, often from groups of investors. The average market capitalization of firms in the LiquiStat sample, however, appears greater than the firms in the other samples.

Table 2 -- Comparative descriptive statistics

	LiquiStat 2005-06	**FMV 80-05**	**FMV 97-05**	**FMV 02-05**	**Finnerty 91-97**	**Bajaj 90-95**
Discount Distribution						
count	61	475	237	91	101	88
mean	32.8%	22.0%	21.6%	14.6%	20.1%	22.2%
standard deviation	14.9%	19.4%	22.4%	19.9%	17.9%	
1st quartile	19.1%	9.9%	9.0%	6.8%	NA	NA
median	34.6%	19.4%[a]	17.8%[a]	12.6%[a]	15.5%	20.7%
3rd quartile	44.0%	33.3%	33.3%	22.7%	NA	NA
Volatility Distribution						
count	61	467	236	90	NA	88
mean	87.5%	93.5%	110.0%	76.5%	NA	74.6%
standard deviation	36.0%	111.4%	144.7%	32.0%	NA	NA
1st quartile	69.1%	58.1%	68.0%	54.8%	NA	NA
median	77.2%	77.2%[n]	88.4%[c]	72.8%[c]	NA	NA
3rd quartile	87.9%	105.5%	120.2%	92.2%	NA	NA
Other Sample Characteristics						
average market cap ($m)	325.3	162.6	188.5	149.3	153.7	117.7
average shares sold/total	0.5%	11.4%	10.2%	11.9%	16.6%	15.9%
average proceeds ($m)	0.7	13.1	15.3	17.7	9.4	13.0

Note: a, b, and c = significant at the 1%, 5%, and 10% levels, respectively. n = not significant at the 10% level.

Given the much higher average discounts in the LiquiStat sample – despite an average time period of restrictions left of only 138 days – we would expect the LiquiStat sample to show significantly greater mean and median volatility numbers than the private placement studies. If we assume that the private placement samples have longer expected restriction periods than the LiquiStat sample, the most likely explanation for the higher

discounts in the LiquiStat sample is higher risk. However, this does not seem to be the case.

As shown in Table 2, the median volatility statistic for the LiquiStat sample is in the range for the FMV volatility statistics, across several samples. Further, based on the Wilcoxon rank sum test, the difference between the LiquiStat and the FMV volatility statistic samples is not significant, even at the 10 percent level, for the time period 1980-2005. The volatility of the FMV sample is significantly lower than the LiquiStat sample for the 2002-2005 period, but only at the 10 percent level of significance (while the discounts are significantly smaller at the 1 percent level) – and for the 1997-2005 period, the volatility of the FMV sample is higher than the LiquiStat sample (again, only at the 10 percent level of significance). As previously noted, the FMV sample, because it is continuously updated, is the only one that covers time periods close to those of the LiquiStat sample. If we can assume a minimum illiquidity period of one year for the restricted stock studies versus only 138 days in the LiquiStat database, and similar volatility levels, the significantly higher discounts in the LiquiStat sample is surprising. Several explanations are possible, including:

(1) The measurement errors associated with the uncertain market reference price for private placements may result in systematic downward biases for the discount.
(2) Factors other than illiquidity may be at play in the private placement process that systematically reduce the observed discount (more than offsetting the additions to the discount from information or other effects). This might include control or other features granted to investors.
(3) The actual expected illiquidity periods for the shares sold in the private placement studies may have been significantly shorter than the 1 year (or 2 years in older studies) Rule 144 illiquidity period.

The last explanation might be the most convincing one. PIPE investments have become highly popular partly because issuers often register the stock shortly after the private placement. When investing, PIPE buyers have fairly strong visibility over how long they

will have to wait for the shares to be registered. However, those details are not always available to the authors of private placement studies. Thus, whether or not stock is issued with registration rights, or even a promise of registration very shortly after the placement, may be unknown. This, if true, would tend to overstate the actual expected period of illiquidity for the shares in the studies. The possibility that the shares in the LiquiStat study are riskier than those in other studies cannot be rejected, of course. There may be aspects of the perceived riskiness of a stock issue that is not captured by the volatility of stock returns. However, based on this evidence, it is more plausible that private placement studies include a too-great fraction of issues where the issuer has promised registration very shortly after the placement. In other words, the average "holding periods" of the private placement studies may be lower than the one (or two) year Rule 144 period and might even be lower than the roughly 4 months of the LiquiStat database.

Cross-Sectional Analysis

The buyers and sellers are anonymous to each other on the RSTN. Also, because buyers and sellers are non-affiliates, they can be assumed to have no material nonpublic information about the issuing firms and to have the same amount of information on average (information symmetry, in the average case). Thus, we can assume that there are no significant information effects on pricing. Likewise, since these transactions result in no new capital and no changes to the ownership structure of issuing firms, capital scarcity and control and monitoring effects can be ignored. Thus, the analysis is limited to variables associated with the value of liquidity.

The period of illiquidity remaining for each block of stock sold was measured as a function of (1) the 356 day holding period necessary to take advantage of Rule 144, (2) the time since the stock was purchased from the issuer, and (3) any evidence that the issuer might register the shares and, thus, allow for resale earlier than expected under Rule 144. Because of the importance of this variable to both buyers and sellers in the restricted stock "market," this information is very well documented and these measures

are assumed to be precise. As will be further explained below, we are using the cubic-root of the number of days remaining of illiquidity ("Root of Days" in Table 3) as our independent variable. The predicted sign of the coefficient is positive.

Stock price volatility ("Variance" in Table 3) was measured as the variance (σ^2) of the continuously compounded return on the stock, measured daily over one year, or since public trading began if less than a year prior to the transaction, and annualized. Stock price volatility is a factor in both the Longstaff and Finnerty models and is logically an important driver of the illiquidity discount. The greater the volatility of a stock, the greater is the possibility for losses during the period of illiquidity (or, alternatively, the greater opportunity for trading gains if the holder is free to trade the stock). Thus, the predicted sign of the coefficient is positive. As an alternative measure of the riskiness of the security, we use two dummy variables for indicating either very low stock prices (less than $2 per share) or very high stock prices (more than $20 per share) which would tend to have a quality-signaling effect to the market ("Low Price" and "High Price" in Table 3). The predicted signs of the coefficient are negative for High Price and positive for Low Price.

The trading volume of the stock is an indication of how difficult it will be to sell the block after the period of illiquidity is over. For most of the transactions in the database, the blocks are not large enough to cause liquidity constraints under Rule 144, which allows for resale after the holding period is over of one percent of the issuer's shares outstanding every calendar quarter. However, even if there are no legal volume limits, if trading volume is too low relative to the block size, it might still limit a block seller's ability to turn shares into cash quickly without impacting the market price. To see if this is an important factor, we have additionally regressed the discount on a dummy variable set to 1 if the block size sold was greater than 5 days' of market trading volume ("Shares-to-Volume" in Table 3). The predicted sign is positive.

Table 3 -- Regression Analysis

variables	*coefficients*	*std. error*	*t (df=36)*
Intercept	0.1308	0.0450	2.905
Shares-to-Volume	0.1250	0.0358	3.492
Low Price	0.0664	0.0339	1.958
High Price	-0.1331	0.0244	-5.451
Variance	0.0484	0.0162	2.985
Root-of-Days	0.0273	0.0083	3.294

Table 3 shows the result of the cross-sectional regression analysis. All of the coefficients have the expected sign. The R^2 is 0.598 and the standard error is 0.099. The F-statistic, at 16.36, is significant at the 1 percent level. The individual t-statistics indicate that the all of the coefficients are significant at the 1 percent level, except Low Price which is significant at the 10 percent level. All t-statistics and standard errors reported are heteroscedasticity-robust.

Discount Time-Decay

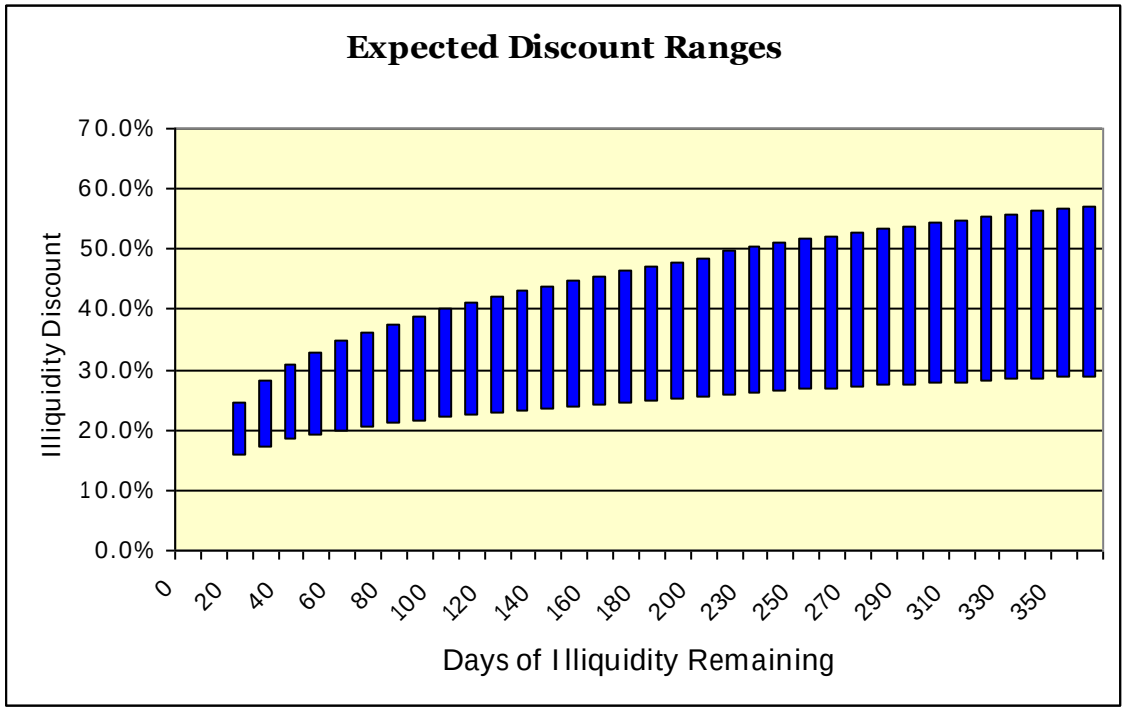
In the sample, the discount shows a significant, positive relationship with the days of illiquidity remaining, consistent with the idea that the illiquidity of the shares is a significant driver of the discount. The data may also be consistent with the findings of Amihud & Mendelson, who found a "clientele" effect where investors who place a high value on liquidity will tend to own short-term and highly liquid securities, while investors who place a lower value on liquidity tend to own less liquid securities.[42] This clientele effect suggests that asset returns may be an increasing, but concave, function of illiquidity. Extended to the marketability discount itself, we would further hypothesize that the discount would be an increasing but concave function of the days left to liquidity.

Running separate regressions of the discount with days remaining, the square root of days remaining, and the cubic-root of days remaining, reveals a better fit with the cubic-root of days remaining. Note that the concavity of the discount-illiquidity period curve (and, presumably, the "clientele" effect) is consistent with the upper-bound formula of

[42] Amihud, Y., and H. Mendelson (1986) Asset pricing and the bid-ask spread, Journal of Financial Economics, 17, 223.

Longstaff, but not with the formula derived by Finnerty. Chart 1 shows the relationship between the period of restriction left and the discount, with the predicted discount within the 90 percent confidence interval.

Chart 1



Pricing Model Tests

As a means of testing the theoretical models, predicted discounts were calculated for each transaction in the LiquiStat database using all three theoretical models. Descriptive statistics for each set of predicted discounts – along with the actual discounts measured – are presented in Table 4 below.

None of the theoretical models performs well against this data-set. The averages of the predicted discounts differ from the average of the actual discounts by more than 20

percentage points in each case.[43] The Longstaff and Tabak models generate several predicted discounts that are higher than 100 percent and almost 100 percent, respectively, which is obviously inappropriate. The two models provide high discounts because of the relatively high level of volatility of the stocks in this sample.[44] However, while higher than the average for public companies, this level of volatility is fairly normal for companies issuing stock privately. This should serve as a note of caution against using these models to estimate discounts (instead of just the upper bound).

Table 4 -- Descriptive statistics

	LiquiStat Actual	Predicted Longstaff	Predicted Finnerty	Predicted Tabak
count	61	61	61	61
mean	32.83%	59.68%	9.13%	57.08%
standard deviation	14.88%	57.75%	8.15%	31.26%
minimum	10.06%	5.96%	0.17%	2.51%
maximum	65.71%	267.13%	36.60%	100.00%
skewness	19.42%	197.74%	123.39%	-5.62%
kurtosis	-90.01%	373.65%	107.03%	-116.67%
1st quartile	19.12%	25.19%	2.66%	32.27%
median	34.62%	37.30%	6.36%	54.01%
3rd quartile	43.96%	69.69%	11.56%	88.83%
interquartile range	24.84%	44.50%	8.90%	56.56%

The output from the Finnerty model is more "reasonable" in the sense that it consistently provides discounts below 100 percent. However, the range indicated by the Finnerty formula is much too low to describe the LiquiStat discounts. The maximum indicated discount is 36.6 percent, which is close to the median of the actual discounts. (Likewise, the top of the interquartile range of the Finnerty model results is below the bottom of the interquartile range of actual results.) The very low discount indications from the Finnerty formula is a reflection of the relatively short periods of illiquidity for the transactions in

[43] In addition, the Wilcoxon rank sum test is used to test the null hypothesis that the distributions of the predicted discounts from the theoretical models are the same as the actual discount distribution. The test results are that the predicted discounts are statistically different from the actuals, in each case.

[44] The fact that the median predicted discount for the Longstaff model, which to some extent eliminates the effect of the large "right-tail" outliers, is much closer to the actual median (closer than the averages) indicates that the Longstaff formula might be more effective at lower volatilities.

the LiquiStat sample. The Finnerty model might be more effective at determining discounts for longer holding-period samples.

8. Summary: Implications for Restricted Stock Valuations

It is quite impossible to disentangle the illiquidity discount from the other portions of the private placement discount and measuring each part separately with any precision. Thus, the myriad of considerations affecting the pricing of private placements raise serious questions about the usefulness of this data for determining liquidity discounts. The lack of precise measurements of the overall discount and the lack of certainty regarding registration rights for private placements are further incentives to develop alternative approaches. However, the data analyzed here does not lend support to the idea that private placement discounts are "too high" to reflect solely the effect of illiquidity. The LiquiStat data may even indicate that private placement discounts are too low to adequately reflect the price of illiquidity. If so, that would be consistent with studies indicating that investors in companies that raise equity privately do poorly. Whether or not this continues to be the case as the LiquiStat database grows will be explored in future versions of this white paper.

The theoretical liquidity discount models may serve as "upper bounds" in some cases and as direct discount estimates in others. Much more study is necessary, but preliminary indications are that the Longstaff and Tabak models should only be used to derive discounts for stocks that are significantly less volatile than those analyzed herein. Also, the Finnerty model would likely only be effective at deriving discounts for significantly longer holding periods than those typical of the dataset analyzed here.

9. Options, Warrants, and Illiquidity

The empirical evidence so far presented on the issue of illiquidity discounts for equity securities may be woefully inadequate – by contrast, the empirical evidence on illiquidity discounts for derivative instruments is non-existent. The LiquiStat database, to the best of our knowledge, is the first attempt at improving this situation. Restricted securities represent a $1.2 trillion asset class, according to the DTCC. Non-traded warrants comprise a significant portion of this total, as does non-traded stock options. According to the PIPEs Report, warrants were issued as "sweeteners" in 56 percent of all common stock PIPE deals (common stock deals represent approximately two-thirds of the $33 billion, 1,800 transactions, PIPEs market) in 2006.[45] The NCEO estimates that approximately 4,000 companies with more than 9 million plan members have option grant plans as of 2006.[46] More accurate valuation methods – which take into account the lack of marketability of these instruments – are needed for accounting, portfolio valuation, and tax purposes, among others.

Depending on the type of option and its payoff characteristics, several theoretical models are used to value traded or otherwise fully-liquid instruments. The standard model for a European-style call option on common stock is the Black-Scholes formula:

$c = SN(d_1) - Ke^{-rT}N(d_2)$, where

$$d_1 = \frac{\ln(S/K) + (r + \sigma^2/2)T}{\sigma\sqrt{T}}, \text{ and}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

[45] The PIPEs Report, Vol. V, No. 1, January 16, 2007.
[46] www.nceo.org/reference/index.html.

The model is based on the following assumptions:[47]

1. The stock price follows a constant Brownian motion (with μ and σ constant).
2. Short selling with full use of proceeds is permitted.
3. There are no transactions costs or taxes and all securities are perfectly divisible.
4. There are no dividends during the life of the option or warrant.
5. There are no riskless arbitrage opportunities.
6. Security trading is continuous for both the option and the stock.
7. The risk-free rate of return is constant and the same for all maturities.

None of these assumptions hold perfectly in real-world situations; however, for fully-liquid stock options on actively traded stocks, the assumptions hold well enough to have permitted the Black-Scholes option model to become ubiquitous in use among options traders. Known biases in the model ("volatility smiles," for example) for actively traded options are typically very minor and can be handled automatically by trading software. With non-tradable options and warrants, however, the discounts from the model price can be expected to be quite significant. As will be further shown below, these discounts are typically greater than discounts for restricted stock.

Option Valuation Concepts

A few more concepts and terms typically found in option and warrant contacts should be introduced before discussing the LiquiStat data on warrants and its implications for non-traded option and warrant valuations:

European, American, and Asian Options. European-style options are exercisable only at the end of the option period, while American options are exercisable at any time during the life of the option. An Asian option is exercisable at the end, but derives its payoff from the average price of the stock during the option period, rather than from the price at the exercise date. A Lookback option's payoff is derived from the maximum (or sometimes minimum) stock price during the life of the contract.

[47] Hull, J. (2006) Options, Futures, and Other Derivatives, 6th ed. Pearson Prentice Hall. pp 290-291.

Intrinsic Value and "Moneyness". In-the-money options have positive intrinsic value, meaning they would yield a profit if exercised (i.e., the stock price is greater than the strike price). An option's value over and above its intrinsic value is called its "time value" (or, because most options have positive time values, its "option premium" or "time premium"). The "moneyness" of an option can be defined as the fraction of its stock price over its strike price (S/K). In practical analysis, since S/K is not a particularly "well behaved" variable, ln(S/K) can be used instead.

Delta. An option's delta is the relationship between the option value and the stock price. It is defined as $N(d_1)$ in the Black-Scholes formula (see above) for the standard call option.

Cashless Exercise. This feature, quite common in both option and warrant contracts, allows the holder to exercise the warrant without paying any cash. The warrant or option is net-settled with stock equal to the total intrinsic value of the warrant or option at exercise. Cashless exercise may also be granted subject to certain conditions, for example a time limit or only after certain criteria are met.

Call-Caps and Barrier Options. Warrant contracts also often have limitations on their exercise. A typical call-cap provision would allow the issuer to redeem the warrant (i.e., force exercise) if the stock price has exceeded 200 percent of the strike price for more than 20 consecutive trading days. Such provisions closely mirror those typical of Barrier options and warrants with call-caps can be valued with standard Barrier option models.

Other Common Terms. Other terms typically seen in option and warrant contracts include anti-dilution provisions, ownership limitations, authorized share failure redemption rights, listing failure redemption rights, change of control rights, registration rights, and transfer restrictions.

Illiquidity and Exercise Behavior

Thus far, most of the work published on illiquid options and warrants focus on the behavior of holders of illiquid stock options (mostly, employee stock options). The conclusions from these studies were important in framing the debate over SFAS 123 and its revised version. Kulatilaka and Marcus note that a holder who wants to reduce his option position would sell part of the position.[48] "Because employee stock options are not transferable, however, the only way to cash them in is to exercise them […] Such early exercise reduces the market value of the options. Kulatilaka and Marcus derive an early exercise model, where early exercise is driven by the need for diversification. Their results imply that historical exercise patterns, since they are driven by past stock price performance, are a poor guide for future exercise patterns. Their results further imply that – although the value of traded options always increases with volatility – depending on the level of investor risk aversion, the value of illiquid options may sometimes *decrease* with increasing volatility (because higher volatility at some point leads to earlier exercise). The latter implication is empirically supported for non-traded warrants by the data in the LiquiStat database.

Hall and Murphy show that executives demand large premiums for accepting stock options in lieu of cash compensation because options are worth less to executives than they cost to the issuing firm.[49] Applying a certainty-equivalent approach, they find that the Black-Scholes model always overvalues non-traded stock options, that far in-the-money executive options are routinely exercised at vesting or fairly shortly thereafter because the expected utility from locking in their gains exceed the utility from holding the options. Their model indicates that executives with low levels of risk aversion and a high concentration of wealth tied up in the company's equity assign values to stock options between 25 and 70 percent of the Black-Scholes value. In fact, in this model, values are in some circumstances assigned below intrinsic value (which in and of itself

[48] Kulatilaka and Marcus (1994) "Valuing Employee Stock Options" Financial Analysts Journal 50 (Nov/Dec) p. 46-56.
[49] Hall and Murphy (2002) "Stock Options for Undiversified Executives" Journal of Accounting and Economics 33 (Feb) p. 3-42.

would tend to explain early exercise behavior). Carpenter also holds that the value to executives of their options can be different from their cost to shareholders.[50] However, her model does not require estimates of wealth concentration or risk aversion, which are unobservable. Based on a sample of 40 executive option grants, Carpenter finds an average *actual* time to exercise of 5.8 years (as opposed to total allowed time to exercise of 10 years).

Finnerty shows that the FASB's chosen method (see below) of shortening the time to exercise cannot fully account for the lack of liquidity for employee stock options and will therefore tend to overstate their fair market values.[51] Finnerty's model solves the problem of how to use historical early-exercise data from an issuer to estimate future early-exercise patterns. One cannot just extrapolate from the past as past exercise behavior has been driven by stock price patterns which may not be repeated. Finnerty notes that since options are leveraged investments, the "impact of any transfer restrictions will be magnified, and the discount for lack of marketability should be greater" for options than for restricted stock – which is supported by our empirical data. Overall, Finnerty finds that employee stock options, at grant, are worth approximately half their Black-Scholes values.

Government Views

Government regulation plays a part in the valuation of stock options for both financial accounting and tax accounting purposes. The standard of value in financial accounting matters is "fair value," while the standard of value in tax matters is "fair market value." Issued by the Financial Accounting Standards Board (FASB) in 1995 and revised in 2004, Statement of Financial Accounting Standard No. 123 provides rules for what constitutes acceptable valuation methods when determining the compensation expense of a reporting company associated with the company's employee stock option grants. SFAS

[50] Carpenter (1998) "The Exercise and Valuation of Executive Stock Options" Journal of Finance 52 (Mar) p. 127-158

[51] Finnerty (2005) "Extending the Black-Scholes-Merton Model to Value Employee Stock Options" Fordham University working paper, January 2005.

123R allows for the use of both closed-form models (such as the Black-Scholes) and methods such as binomial trees or monte-carlo simulation. The main difference between an SFAS 123R valuation and a valuation appropriate for portfolio valuation and/or tax valuation purposes is that SFAS 123R specifically disallows the application of an illiquidity discount. Rather, the accounting standard requires that companies estimate the "effective" time to exercise of the options granted, based as much as possible on actual early-exercise behavior of plan participants.[52] This is a reasonable enough method for compensation expense determination purposes, but would not be appropriate for valuing non-employee options or for performing valuations for other purposes (including tax purposes).[53]

The Internal Revenue Service has provided a revenue procedure, No. 98-34, as a "safe harbor" for valuing stock options. The procedure requires the use of the Black-Scholes or binomial models and places certain limitations on the specification of the inputs to the models. Most importantly, once cannot claim conformance with Rev. Proc. 98-34 if discounts are taken for the illiquidity of the options valued. Such revenue procedures are typically issued to remedy perceived taxpayer "abuses," in this case abusively low valuations of stock options. However, as will be clear from the evidence presented below, the Service may have overreached in this case, as the application of Rev. Proc. 98-34 will almost always lead to severe overvaluation of non-traded options or warrants.

[52] The FASB's position is based on the empirical research (noted above) on early-exercise patterns. It remains to be seen whether or not Finnerty's model – which more accurately reflects the fair market value of the options granted – will be accepted in financial statements under SFAS 123R.

[53] Some of the difference in methods is due to different value premises: the difference in focus between estimating the cost, to the issuer, of the services received from its employees or others and the fair market value to the option grant recipient – i.e., what the options would trade for, including all their restrictions, in an arm's-length trade between a willing buyer and willing seller, neither of whom being under any compulsion to trade.

10. Empirical Analysis – Warrants

Transaction Sample

The data analyzed herein is from the LiquiStat™ database of private sales transactions on the Restricted Securities Trading Network. LiquiStat and the RSTN have been described above (see Section 7). The sample analyzed in this version of this white paper included 76 transactions in non-traded warrants, exercisable for shares of publicly traded companies. Table 5 below provides a description of the warrant sample. For each transaction in the database, the theoretical model value of the warrant was computed, which is the liquid-equivalent value of the warrant (assuming that both the warrant and the underlying stock are fully liquid and that all of the other assumptions of the Black-Scholes model hold).[54]

Table 5 – LiquiStat Warrant Trades

	Intrinsic Value	*Moneyness*	*Time to Expiration (Years)*	*Market Price*	*Volatility*	*Black-Scholes Discount*
Mean	$1.10	0.15	3.3	$5.39	75.6%	41.5%
Standard Deviation	1.47	0.42	1.2	4.99	25.4	18.0
Minimum	0.00	-0.99	0.1	0.40	43.9	1.8
Maximum	6.76	1.72	5.0	21.12	178.5	71.4
Medians – Data Sorted by Magnitude of Discount from Black-Scholes Value						
1st Quintile	$1.68	0.33	2.2	$4.75	54.9%	16.1%
2nd Quintile	1.30	0.24	3.6	4.66	72.5	33.5
3rd Quintile	0.26	0.00	3.5	5.50	68.8	44.0
4th Quintile	0.25	0.04	4.0	5.75	82.8	54.4
5th Quintile	0.00	-0.10	3.3	1.95	93.5	61.5

[54] For simplicity, the differences between actual and model prices are referred to herein as "illiquidity" or "marketability" discounts. The discounts may in fact represent any number of divergences from the theoretical model values in addition to just liquidity issues (for example, the Black-Scholes formula may consistently overvalue warrants with long time to expiration). However, we believe that the lack of marketability for these securities is the cause of the majority of the discount. The goal, of course, is not to separate between various elements of the warrant discount, but to arrive at workable valuation models for non-traded warrants. This we can do without differentiating between the various causes of the discount.

It is immediately apparent from the table that the average discount is higher than for restricted stock and that the range is wider. For options and warrants, the illiquidity discount can be expected to vary significantly, depending on how far in the money the warrant is. This is because the intrinsic value of the warrant can often be immediately realized by the holder by exercising early. Only the time-value portion of the warrant's value requires a holding period. Thus, illiquidity is a much more severe value detriment for out-of-the-money options and warrants. As shown in Table 5, the average intrinsic value is significantly higher for warrants in the low-discount quintiles, while the average moneyness is significantly lower for warrants in the high-discount quintiles.

Other Discount Indications

In the LiquiStat database, illiquidity discounts are calculated from both (a) the theoretical option value and (b) the calculated theoretical time value of each warrant, calculated by deducting the intrinsic value of the warrant from its full theoretical value. The time-value discounts are very substantial when compared with the discounts from full Black-Scholes value. In the LiquiStat database, time-value discounts range from 20-30 percent at the low end to more than 100 percent at the high end (certain warrants can sell at below their intrinsic values). The median time-value discount is 61 percent, for the 76 warrant trades analyzed herein.

The LiquiStat database also provides discount indications from model values that incorporate valuation models other than the Black-Scholes method. The Black-Scholes method, for example, cannot properly calculate the value of warrants with call caps. The best theoretical ("as if fully-liquid") valuation model for a warrant with a call cap of B (with a rebate at redemption of B-K) is the standard pricing model for an up-and-out barrier option with a zero dividend yield:

$$c = SN(x_1) - Ke^{-rT}N\left(x_1 - \sigma\sqrt{T}\right) - S\left(\frac{B}{S}\right)^{2m}\left[N(-y) - N - y_1\right] + Ke^{-rT}\left(\frac{B}{S}\right)^{2m-2}\left[N\left(-y + \sigma\sqrt{T}\right) - N\left(-y_1 + \sigma\sqrt{T}\right)\right]$$

, where

$$y = \frac{\ln\left(B^2/SK\right)}{\sigma\sqrt{T}} + m\sigma\sqrt{T} \qquad m = \frac{r + \sigma^2/2}{\sigma^2}$$

$$x_1 = \frac{\ln\left(S/B\right)}{\sigma\sqrt{T}} + m\sigma\sqrt{T} \qquad y_1 = \frac{\ln\left(B/S\right)}{\sigma\sqrt{T}} + m\sigma\sqrt{T}$$

The model yields a lower value than the Black-Scholes formula (the call cap reduces the value of the warrant). The discounts from the barrier-option values tend to be somewhat lower on average than the discounts from the full Black-Scholes value for options without call caps. This may indicate that investors do not believe the issuer will exercise its option to redeem the warrants even if the call cap is significantly exceeded. Thus, call cap warrants may be valued using a direct comparison with discounts for other call cap warrants, or with a regression analysis of the entire LiquiStat database, with a dummy variable or other signifier to measure the effect on the discount from the Black-Scholes values due to the presence of a call cap provision.

11. Summary: Implications for Warrant and Option Valuations

The LiquiStat database represents the first-ever study of real-world empirical data on transactions in non-traded options and warrants. The discount indications indicate significantly higher discounts than those normally applicable for restricted stock in similar companies. This may have two main reasons: options and warrants are more "leveraged" investments than shares, in the sense that, at least at certain levels of moneyness, a certain percentage appreciation in the price of shares will lead to a greater percentage appreciation in the price of the option. The other main reason is that the holding periods for options and warrants, i.e., the period required to get the full benefit of their time values, are longer than the 1-year holding period under Rule 144.

The time value discounts are significantly higher than the discounts from full Black-Scholes value. Time value discounts in the LiquiStat sample tend to be approximately 1.5 times the full value discounts. Discounts are greater the longer the time to expiration, the higher the volatility, and the further out of the money an option is.

Both the existence and the size of the discounts for non-traded options and warrants have been predicted and analyzed in prior theoretical papers. The LiquiStat data, as presented in this white paper provides confirmation for the fact that investors will not pay full Black-Scholes values for non-traded warrants and confirms that the discounts are substantial. The results herein are also consistent with prior theoretical papers in terms of the direction of the discounts due to variations in variables such as the time to expiration.

The FAS123R method may in most cases overvalue issued employee stock options. Because the method explicitly rejects the application of a liquidity discount, and the method of "shortening" the time to exercise may not fully reflect the effect of illiquidity, the method has a tenuous relationship with the "true" market value of issued options. The method specified as providing "fair market value" of options for tax purposes, Revenue Procedure 98-34, on the other hand, is always and everywhere inappropriate.

No non-traded options or warrants would sell at full Black-Scholes value, using volatility inputs from the market and the full time to expiration. The discounts reported herein are in reality discounts from the Revenue Procedure 98-34 values. The very significant size of these discounts should be very strong evidence against using this method, as it very dramatically overvalues non-traded options.

12. Future Directions

The RSTN is growing rapidly. More than a dozen new transactions are concluded every month, as of the writing of this draft of this white paper. Therefore, our ability to provide more detailed analyses of the impact of market conditions, events, contractual features, financial and operating conditions of the issuer, and any other important characteristics relevant to the value of illiquid shares, warrants, and options will increase rapidly. The lower degree of "noise" of the LiquiStat sample makes it uniquely well-suited to this purpose. Future updates of this white paper will provide additional analysis on larger samples. We expect a greater proportion of higher-quality paper in future samples.

Questions or Comments?

Please contact us at:

Pluris Valuation Advisors LLC
61 Broadway, Suite 1000
New York, NY 10006
P 212.248.4500
F 212.248-4599
www.pluris.com

Espen Robak, President

erobak@pluris.com

Pluris
Valuation Advisors LLC

The Key to Accurate Valuation of Illiquid Assets

How do you value assets when there is no public marketplace for them?

You'll need a valuation firm with in-depth experience and access to the most up-to-date real-world data. Pluris Valuation Advisors was created to meet this need.

Pluris specializes in valuing illiquid assets, such as restricted securities and stock options. Valuation needs Pluris can help fill include:

Portfolio Valuations.
Pluris values restricted stock, warrants and other illiquid securities for investment banks, hedge funds and other investment funds, or other reporting entities.

Financial Reporting Valuations.
Pluris provides valuations of stock option grants and non-vested shares of reporting entities, as well as valuations of complex derivatives for disclosure and accounting purposes.

Tax Valuations.
Pluris values illiquid securities and other assets for estate tax, gift tax and income tax returns.

Transaction Opinions.
Pluris provides fairness and solvency opinions needed as part of the due diligence process for corporate transactions.

Litigation Support.
Pluris provides shareholder litigation support services when expert assistance is required.

Using a professional valuation firm can help ensure that you have valuations that are not only fair and accurate, but defensible.

Pluris is the key to all of your illiquid-asset valuation needs.
Contact Pluris today at **212-248-4500** or info@PlurisValuation.com.

The LiquiStat™ Database

The more data from real-world transactions a valuation is based on, the better you can support your opinion of an asset's value.

Pluris Valuation Advisors created the proprietary **LiquiStat™** database of restricted stock transactions for this purpose. We believe our database provides the most in-depth, comprehensive empirical transaction data available on restricted securities transactions.

The **LiquiStat™** database is based on data licensed from our affiliate, Restricted Stock Partners, which operates the Restricted Securities Trading Network (RSTN), a proprietary network of institutional and accredited investors interested in buying and selling restricted securities.

Pluris Valuation Advisors LLC
17 Battery Place, 11th Floor
New York, NY 10004

(212) 248-4500 Phone
(212) 248-4638 Fax

info@PlurisValuation.com

www.PlurisValuation.com

How the Restricted Securities Trading Network Operates

Despite their restricted status, there are exemptions holders can use to sell their restricted positions.

How do these privately negotiated sales work?

The process includes the following steps:

1. List the Position

Owners of restricted securities contact RSP and indicate the position they would like to sell. A Restricted Securities Specialist gathers basic information, such as the ticker symbol, acquisition date of the securities, how the securities were acquired and the nature of the restrictions on them.

2. Obtain Bids

RSP contacts potential buyers based on their indicated preferences, which may include the size of the position, liquidity of the underlying public stock, the industry the company issuing the securities operates in and other factors. The seller may limit the amount of information RSP provides to potential buyers.

3. Negotiate Terms

Once one or more suitable buyers are identified, the Restricted Securities Specialist helps the parties negotiate key terms of the transaction. All parties remain anonymous.

4. Execute a Restricted Security Order

When both parties agree on terms, the buyer and seller execute a Restricted Security Order (RSO), which details the transaction. The document is exchanged between buyer and seller, and identities are revealed.

5. Closing

RSP facilitates all aspects of closing, including preparation of the following documentation:

- Purchase agreement
- Buyer's representation letter
- Seller's representation letter
- Opinion letter from counsel
- Escrow agreement

Once the documentation is finalized and executed, the opinion letter, the physical securities and other transfer documents (e.g., stock power) are sent to the company's transfer agent, along with a letter requesting that a stock certificate be reissued in the name of the buyer.

If the documentation is in order, and the issuer has approved the transfer, the transfer agent cancels the existing stock certificate and issues a new certificate. During this process, the buyer's cash payment is typically held in escrow pending completion of the transfer.

Generally, once the RSO is executed, the buyer will receive a new certificate and the seller will receive proceeds within two to three weeks.

No fees are charged for listing or bidding on restricted securities for sale. Commissions are fully disclosed and are not charged until completion of the transaction.